Piaget
and collaborators
on
Possibility and Necessity
And the relevance of/to AI/Robotics

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This is a very personal introduction to, and appraisal of some aspects of Piaget's two last books (Piaget & *others*, 1981) (Piaget & *others*, 1983)

I suggest a "rational reconstruction" of Piaget's theories, showing how his work might be assimilated into future mainstream AI – transforming both developmental psychology and AI.

These slides (liable to ongoing development) are available in my 'talks' directory:
http://www.cs.bham.ac.uk/research/projects/cogaff/talks/#piaget

A related presentation will be given at AGI 2011: http://agi-conf.org/2011/

Piaget's two last books

NB: Each has several additional co-authors.

Piaget, Jean, et al.,
Possibility and Necessity Vol 1. The role of possibility in cognitive development,
U. of Minnesota Press, Tr. by Helga Feider from French in 1987, (Original 1981)

Piaget, Jean, et al.,
Possibility and Necessity Vol 2. The role of necessity in cognitive development,
U. of Minnesota Press, Tr. by Helga Feider from French in 1987, (Original 1983)

As far as I know there has been no development of the ideas in these two books since their publication.

Before I learnt about these books, I consulted several well known researchers in developmental psychology about some of the research problems mentioned below. None mentioned Piaget's books – until I asked Annette Karmiloff-Smith, to whom I am very grateful for the pointer.

Reading these books may be hard for people unfamiliar with Piaget's work or developmental psychology – though mathematicians and physicists may find it easier than others.

An older book which I found very inspiring many years ago was partly stimulated by Piaget's work:
The Child's Discovery of Space: From hopscotch to mazes – an introduction to intuitive topology.
Sauvy and Sauvy (1974)

Also relevant are autobiographical anecdotes by mathematical colleagues reported in Borovik (2010).

I may later add other links to relevant online material on Piaget. Suggestions welcome

A useful review

Both books were reviewed by Leslie Smith in 1994 (Smith, 1994).

He writes at the end (I've broken up the paragraph):

> "In sum, *Possibility* and *Necessity* are required reading for probably all developmentalists. They are inspirational texts.
>
> If the author was a novice, with youth on his side, these texts would herald future promise.
>
> They are still inspirational, written as they are by a master near the end of his life, heralding future promise for Piaget's theory, which continues to develop."

(If there are other freely available online reviews, please send me links.)

Abstract posted before the talk on 21st Feb 2011

It is not widely known that shortly before he died Jean Piaget and his collaborators produced a pair of books on Possibility and Necessity, exploring questions about how two linked sets of abilities develop:

(a) The ability to think about how things might be, or might have been, different from the way they are.
(b) The ability to notice limitations on possibilities, i.e. what is necessary or impossible.

I believe Piaget had deep insights into important problems for cognitive science that have largely gone unnoticed, and are also important for research on intelligent robotics, or more generally Artificial Intelligence (AI), as well as for studies of animal cognition and how various animal competences evolved and develop.

The topics are also relevant to understanding biological precursors to human mathematical competences and to resolving debates in philosophy of mathematics, e.g. between those who regard mathematical knowledge as purely analytic, or logical, and those who, like Immanuel Kant, regard it as being synthetic, i.e. saying something about reality, despite expressing necessary truths that cannot be established purely empirically, even though they may be initially discovered empirically (as happens in children).

It is not possible in one seminar to summarise either book, but I shall try to present an overview of some of the key themes and will discuss some of the experiments intended to probe concepts and competences relevant to understanding necessary connections.

In particular, I hope to explain: (a) The relevance of Piaget's work to the problems of designing intelligent machines that learn the things humans learn. (Most researchers in both Developmental Psychology and AI/Robotics have failed to notice or have ignored most of the problems Piaget identified.) (b) How a deep understanding of AI, and especially the variety of problems and techniques involved in producing machines that can learn and think about the problems Piaget explored, could have helped Piaget describe and study those problems with more clarity and depth, especially regarding the forms of representation required, the ontologies required, the information processing mechanisms required and the information processing architectures that can combine those mechanisms in a working system – especially architectures that grow themselves.

That kind of computational or "design-based" understanding of the problems can lead to deeper, clearer specifications of what it is that children are failing to grasp at various stages in the first decade of life, and what sorts of transitions can occur during the learning. I believe the problems, and the explanations, are far more complex than even Piaget thought.

The potential connection between his work and AI was appreciated by Piaget himself only very shortly before he died.

During final preparation of the talk I added the notion of an exploration domain – a collection of phenomena that can be explored empirically (or in the case of abstract domains, like number theory, or the game of chess, can be explored "quasi-empirically"), and which later can be reinterpreted by reorganising what has been learnt into a "generative system" within which various things can be derived without having to be discovered empirically.

Extensions to the abstract: topics added later

The key idea, which refines things I've written previously about development of mathematical competences, is that the world contains a vast variety of "Exploration domains" (sometimes called "Micro-worlds" in AI, or "Microdomains" in Karmiloff-Smith (1992)) that a child or animal can get to know through play, exploration, observation, and sometimes social interaction.

Humans, and some animals, and cultures, e.g. scientists, seem, after a while, to reorganise what they have learnt in a domain into something like a deductive system that supports inference to new cases that therefore do not need to be tested empirically – and the previous empirical discoveries can also be re-interpreted as "theorems". (This process is generally unconscious, as are the results.)

A well known example is the transition from use of learnt patterns to use of generative syntax in language learning. That's a special case, which I think evolved after similar things had been happening in connection with many other domains in human and non-human learners (Sloman, 1979, 2008a).

However, the processes of exploration by young learners, scientists and mathematicians are not infallible (compare Lakatos (1976)) and so children (and mathematicians) can make mistakes of many sorts in their initial empirical explanation and in their rational reconstruction or in applying their reconstructed systems. Many of the phenomena that Piaget discovered illustrate this.

Also children explore different domains in parallel, though, for various reasons, some cannot be begun until after others have been "done" – e.g. meta-domains. Domains can also be merged to form larger domains.

Even unrelated domains can be co-embedded in a space-time framework; so learners can discover composite domains – e.g. blocks plus plasticene, toy dolls plus toy clothes, sand plus water, etc. And the process of combining domains can introduce bugs into previously acquired competences.

Each child almost certainly has a unique trajectory through this space of explorations, reorganisations, compositions, etc. and that can explain the diversity of experimental phenomena Piaget and colleagues discovered, along with cultural and minor or major genetic differences (autism, Down syndrome, Williams syndrome, blindness, deafness, palsy, etc., etc.)

Three sub-topics

There are three huge topics that I'll only be able to dip into in a very shallow way:

1. The theoretical viewpoint that drove Piaget's experiments on developing understanding of possibility and necessity. (I offer a rational reconstruction.)
2. The experiments themselves and what came out of them. (I report only a tiny sample.)
3. What conclusions Piaget drew and whether there's a better way of thinking about them, not available to Piaget, (I offer suggestions: I suspect he, and Kant, would have approved.)

What Piaget (and his colleagues) wrote about (1), (2) and (3) is hard to understand if you are not immersed in the genetic epistemology school – as I am not.

So I may be misrepresenting him when I relate his ideas to "Exploration domains" below.

I believe his ideas are closely related to the differently formulated ideas I have been developing in ignorance of these books. E.g. in Sloman (1962, 1968/9., 1971, 1996, 2010), and

`http://www.cs.bham.ac.uk/research/projects/cogaff/talks/#toddler`

I shall present my own paraphrase of Piaget's background assumptions – largely based on features of the environment that are potentially relevant to many different biological species – independent of their morphology and sensorimotor mechanisms.

Those features are also potentially relevant to intelligent robots.

SHOW MOVIES – Compare: BigDog, Parrot, Babies, Toddlers.

BigDog (Boston Dynamics) is a very impressive robot, but as far as I know has no conception of what can happen, what could have happened, what must happen, or what it could have done but did not do, or why.

Relevance to other subjects

I think these topics are largely ignored by people working in AI/Robotics, philosophy of mathematics, philosophy of language (e.g. the nature of modal constructs), and related fields: and that's a serious limitation.

Any generalisation I make about work not done has exceptions!

I don't know how many developmental psychologists work on these topics, and whether there are already results, unknown to me, that go beyond what Piaget achieved (concerning Possibility and Necessity).

NB: my exposition does not closely follow Piaget's own. I am offering a "rational reconstruction", which I hope is in the spirit of what he was trying to do, namely, understanding development as a biological process with some (hard to specify) invariant features that come largely from features of the environment in which organisms have evolved. (Compare also McCarthy (2008).)

Personal note: before I learnt about Piaget I was thinking about some of the issues while working on my DPhil thesis (1962) trying to defend Immanuel Kant's philosophy of mathematics against the then widely held opinion that there are only two kinds of knowledge: empirical knowledge and essentially trivial (analytic) knowledge. Kant offered mathematics as demonstrating the possibility of non-empirical knowledge of necessary truths that are not purely definitional or logical. I think Piaget agreed with Kant.

I started learning about Piaget in the 1960s and was probably deeply influenced by his work even though I mostly continued on my own track, especially after learning about AI around 1969 – and decided that designing and testing increasingly complex working fragments of human (and animal) minds was the best way to pursue many old philosophical problems. (Kant would have loved the opportunity to test his ideas.)

Specifying what needs to be explained

(What an engineer would call requirements analysis)

A great deal of work by Piaget (and many studying development of cognition) involves experiments to discover what needs to be explained.

Popper posed a problem about doing this: **you cannot just observe – without some theory providing concepts and questions.**

(Popper, 1934). For an introduction see (Magee, 1985)

How observations are described will depend on what the background theory is.

It follows that as scientific theories develop, including producing new ontologies, we often find better ways of presenting what was previously discovered – and then, what needs to be explained is changed.

Piaget's theories and his descriptions of experimental results make use of many concepts that are quite difficult to understand

E.g. accommodation, assimilation – two of the easiest ones), equilibration, conservation, reversibility, integration, horizontal and vertical decalage, pre-operational, concrete operational, formal operational, structure, ...

I believe Piaget's good ideas can be re-interpreted in the light of the idea presented below that intelligent animals find and investigate **often in parallel** many distinct but combinable "exploration domains" and that their learning has several phases including

(i) finding re-usable patterns (associative learning), and then later
(ii) reorganising the material into a generative (proto-axiomatic) system that allows a learner to work out what can happen or must happen in novel situations. (A possibility ignored in (Gibson & Pick, 2000)).

Getting “behind” Piaget’s work.

I’ll start with the phenomena I think Piaget was trying to explain– since I have been exploring many of the same problems,

e.g. (Sloman, 2008b).

I’ll present a few examples of domains (“exploration domains”) that a thinker (young or old) can explore to find hidden structure, where some of the generalisations may be discovered empirically, then later understood as having a kind of necessity.

Some of the examples come from Piaget – others not, though all have features appropriate for use in Piagetian experiments.

I don’t know whether I am merely repeating suggestions already made by Piaget or some of his followers.

Karmiloff-Smith (1992) suggests that Piaget’s theories downplayed the contribution of the environment in evolution, and that he was committed to a uniform (innate) developmental mechanism for all domains even though the details of each domain are different. I am not sure this is an accurate characterisation of Piaget’s position, though I may be hallucinating my own ideas onto his work.

Features of exploration domains

The “exploration domains” described later are of different kinds, but all are domains of activity that can be based on features of the environment

(though some can also be explored in an abstract manner detached from the environment).

- play/experimental/exploration revealing types of occurrence, and correlation patterns.
- discovery of abstractions (e.g. “conservation laws” or “invariants”) across examples;
- later simplification by (usually unconscious) invention of new principles underlying patterns discovered:
  1. Finding/creating a more powerful ontology specifying what kinds of states, entities, events, processes and causal interactions can be thought about, perceived, predicted, hypothesised,
  2. Extending the set of kinds of matter (physical stuff) that can occur in those events, processes, etc.;
  3. Discovering new constraints on possibilities and necessary connections between possibilities;
- Combining two independently explored domains to form a richer domain,

  for example learning to play with mud, learning to play with wooden blocks, then learning to play with the combination. (Meccano involves several different sub-domains.)

In some cases there is a meta-cognitive transition that is a precondition for taking part in Piaget’s experiments:

- development of meta-cognitive abilities to think about, discuss and answer questions about the things learnt in an exploration domain.

  (Compare Karmiloff-Smith’s “representational redescription” of “microdomains”)
  Can any non-humans can do anything similar?

Exploration Domains are of different kinds

Here are some of the ways in which exploration domains can differ:

- Some contain discrete structures and processes, others a continuum of possibilities.
- Some involve human language, others non-verbal forms of representation.
- Some of the discoveries are about properties of particular kinds of matter or particular material objects, while others are about more abstract entities relevant to many very different concrete physical situations. e.g. Euclidean lines, areas or volumes.
- Many of the examples involve causal connections between events, processes and structures, where
  - some are deterministic (e.g. moving one end of rigid centrally pivoted rod down causes the other end to go up, and vice versa.)
  - others are statistical, e.g. the proportion of ways of getting a total of 5 when throwing two dice.

This is not an exhaustive classification.

I shall start with a relatively simple discrete, domain which leads to an abstraction that is applicable to many different sorts of things that can be ordered.

A wide range of examples can be found in Piaget's publications and also in Sauvy and Sauvy (1974).
A domain relevant to gaining insight into negative integers is described in (Liebeck, 1990).

Computer programming is an ideal way of exploring a wide range of domains by modelling or simulating the structures and processes.

"Turtle programming" in LOGO was one of the earliest examples illustrating this.

Several more examples of computational exploration of non-computational domains are illustrated in
`http://www.cs.bham.ac.uk/research/projects/poplog/examples`

Explorations of (discrete) possibilities: Order

This exploration domain can use physical objects (cards or blocks with letters) or can be treated as a collection of thought experiments:

A B C D E

If you can repeatedly swap any two adjacent elements you can change the order of letters, e.g. to this using one swap:

B A C D E

Can the next transition be achieved by combining adjacent swaps? How?

A B C D E —> E B C D A

Can you go in the reverse direction using adjacent swaps – and if so, how?

How many different sequences of states ("routes") are there between the two states with "A" and "E" swapped? How do you know?

Could there be infinitely many different "swap-routes" between two states?

Is it possible to reverse the whole array using a sequence of swaps?

Order transformations and necessity

Consider the previously presented "swap game", in which cards with letters are presented in a certain order and a target ordering is presented.

The game is played by trying to get from the initial state to the target state by a sequence of pairwise swaps.

A child playing the game may or may not notice various things (and ask "why?"):

- Any card can be moved to the right end by a sequence of swaps, leaving the other cards in order.
- Similarly any card can be moved to the left end.
- The cards at both ends can be made to change position.
- Any two cards can be made to change position.
- Given a row of cards, and another row of similar cards in a different order, the first can be transformed into the second.

  Intermediate stages of comprehension: learning this for specific pairs of rows, then learning it for any two rows of 5 cards, two rows of 6 cards, two rows with any number, etc.
- The procedures work even if some cards in the rows are repeated: e.g. "A B B C" can be transformed to "C B A B"
- The transformation is impossible if one row has more occurrences of a certain letter than the other (e.g "A B B C" "A B C C"
- The colour and size of the letters, the material of which the cards are made (and many more features of the situation) are not relevant to whether the game can succeed or not. (Why not?)
- Would what is learnt in this game be relevant to a game in which there is a collection of nested 2-D figures, e.g. a square inside a triangle inside a circle inside a pentagon?
  What would the "swap" operation become?
  Which of the generalisations true of card-swapping could be applied to the nested figures?
  Why is it a mistake to assume this necessarily involves use of metaphorical reasoning?

A Possible AI project

A possibility for meta-exploration could be undertaken by someone interested in child development who also has AI programming experience.

That could be done by exploring different ways to program a child-like simulated explorer that is capable of:

- perceiving and describing letter sequences
- performing the swap operation on any two items in the sequence
- combining swap sequences, e.g. by writing “virtual programs” to swap virtual cards, and then running the programs.
- noticing patterns that occur when such programs run
- noticing that some patterns never occur

  (e.g. the initial state is never restored after an odd number of swaps)
- reasoning about why things happen or why things do not happen
- examining processes and realising that they have features that are general to a class of processes
- noticing structural mappings between structures, between processes, and between whole domains.

A psychologist exploring such a simulation might notice obstacles to learning, and patterns in how learning occurs in this domain.

This could inspire research into (a) how well some or all children fit such a model and (b) whether brains have mechanisms that could be used for these purposes and (c) how many other animals have such competences and meta-competences, and (d) how such competences evolved, and how many times.

More examples of exploration domains

The examples given so far, involving rows of cards and nested figures, with a "swap" operation are discrete exploration domains: At the lowest level there is a discrete set of entities whose relationships can be changed when processes occur. But there are other sorts.

- People with mathematical minds will quickly realise that the domains presented so far are very much richer than I have indicated, and can be used to explore arithmetical notions, permutations, and other structures.
- Other domains are not discrete but continuous, permitting gradual, and even indiscernible, transitions between states

  (like the "invisible" movement of the hour hand of a mechanical clock).
- Some are a mixture of continuous and discrete – like the domain of movements and configurations of a fixed, discrete set of blocks, meccano pieces, or Lego blocks.
- Some are linear (one-dimensional – totally ordered), some two-dimensional (e.g. embedded in a plane, or in the surface of a sphere), some three or higher dimensional.

  Mathematicians have happily explored infinite dimensional domains.
- Some domains are fixed, static structures whose properties can be explored, while other domains of exploration include events and processes, which themselves may be either discrete, or continuous, or a mixture (think of various kinds of board game).
- Some domains include different kinds of matter – e.g varying in hardness, rigidity, elasticity, fluidity, stickiness, absorbency, solubility in water, electrical or thermal conductivity, and many more – children explore them long before they study physics at school.

Naturally occurring exploration domains

The swap domain may or may not turn up in the life of a child: I chose it as an example likely not to have been encountered by most people.

Anyone who has studied sorting algorithms in computer science, will instantly recognise it, however.
http://en.wikipedia.org/wiki/Bubble_sort

An exercise: try to find or think up, naturally occurring exploration domains that can turn up in the life of humans or other animals.

Consider exploration domains relevant to different nest-building birds, for example?

- Nests made by pushing plant matter and perhaps mud or sand along the ground.
- Nests made by bringing discrete objects, e.g. pebbles twigs, which have to be placed to form a rigid structure.
- Nests made with an outer casing and a softer internal lining.
- Nests made by repeatedly bringing a malleable substance such as mud, portions of which can be pushed together, later forming a rigid structure, after drying.

Are there identifiable separate kinds of expertise required for sub-domains that have to be learnt separately, within a nest-building domain?

Contrast different exploration domains in nests on the ground, nests resting on branches in trees, nests suspended from branches, water-edge nests.

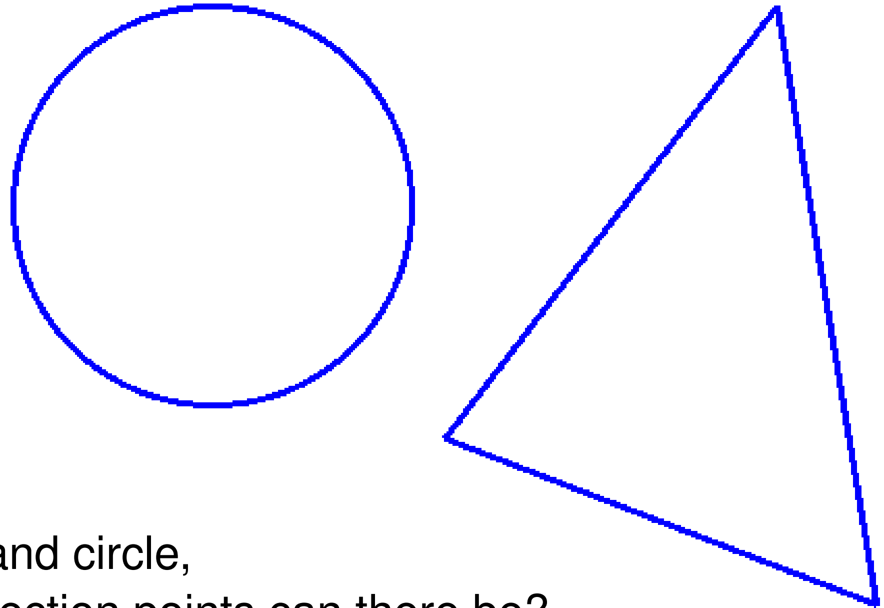
Some species of animals (often called "precocial") seem to be born, or hatched, with all the expertise they need, while others (often called "altricial") have to build their own competences.

How can a genome encode the information required for either of these?
(Sloman & Chappell, 2005; Chappell & Sloman, 2007)

Compare food-obtaining, and food-consuming domains, for herbivores, carnivores, multivores.

Explorations in a continuous domain: Geometry

There many continuous exploration domains with very different properties.
This is a simple example, related to some of Piaget's examples.



Where else could the circle and triangle have been placed on the surface you see?

In how many different locations could the circle have been placed?

Can locations be counted?

What would happen if the circle were to slide to the right, passing through the triangle?

If you alter positions and sizes of the triangle and circle, how many different possible numbers of intersection points can there be?

Could there be nine points of intersection? If not why not?

Does the answer depend on the shape of the triangle?

What if the triangle were replaced by a square, or the circle by an ellipse?

Would the answers change if the lines had a different colour?

What difference could the thickness of the lines make?

Continuous domains can be discretised in different ways, for different purposes.

Exploration is not infallible

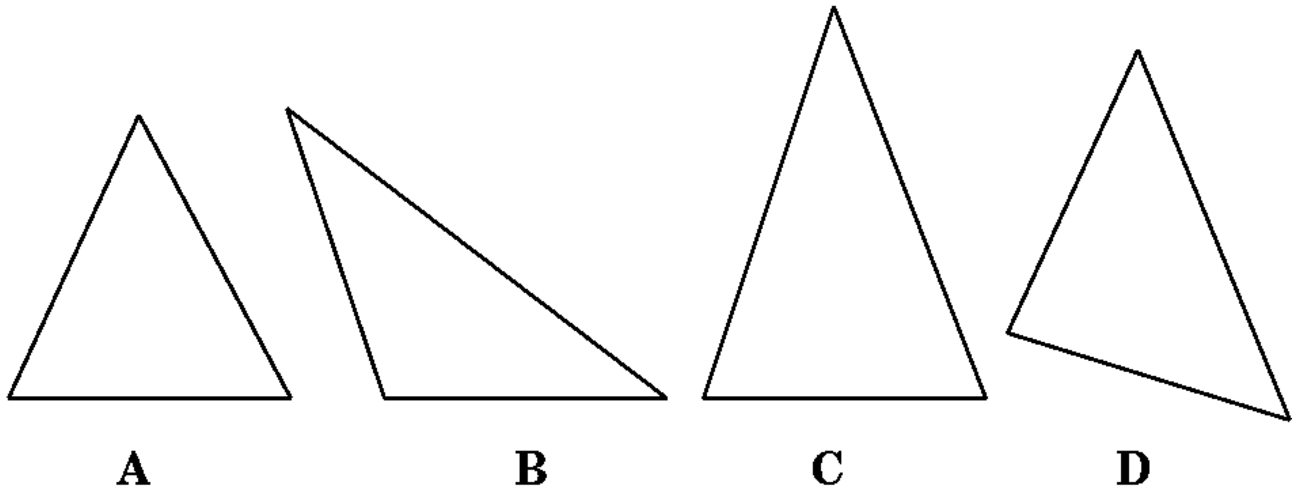
Not only is it possible to make discoveries in an exploration domain: it is also possible to make mistakes by over-generalising on the basis of an observed pattern.

- For example, someone who tries a limited range of experiments in the swap game may wrongly conclude that if more than six swaps occur in a row then an object must change direction of movement, then later discover that that is false, if the swaps are applied to different objects.
- Likewise, someone who considers only a circle that is very much larger than the triangle may wrongly conclude that it is impossible for a triangle and circle to have more than two, or more than three points of intersection.
- Someone playing with the triangle and circle who moves them only in quite large steps may never notice the case where a vertex of the triangle lies on the circumference of the circle, so that two sides share a point of intersection with the circle,
- It is also possible to miss the case where one or more of the triangle sides meets the circle at a tangent, with only one point of intersection.

A failure to encounter certain possibilities, because the exploration does not include enough generalisation can also prevent the learner acquiring certain concepts, such as the concept of a line being a tangent to a circle, or the concept of a line ending on the edge of the circle.

How many of the errors and confusions shown by children in Piaget's experiments can be explained by models of simulated learners that do not explore enough cases, or form incorrect generalisations? (Compare Seely-Brown's BUGGY program, modelling and diagnosing children's arithmetic errors.)

Exploring continuous deformation of triangles



If the triangles are made of rubber bands, held in place by pins what happens to the angles if you move the corners? E.g.

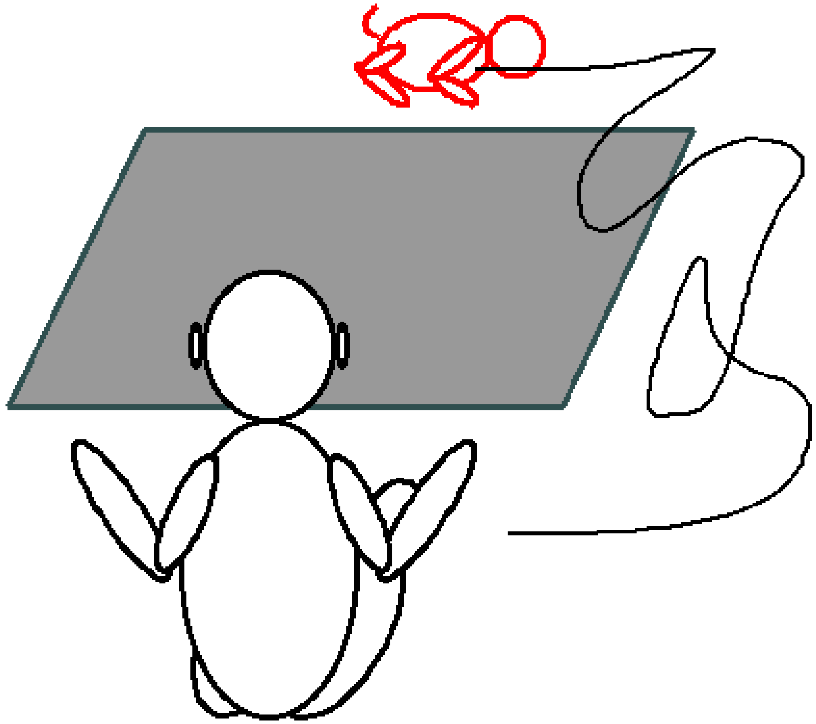
- If you slide the top vertex left or right, parallel to the base, as in **B**?
- If you start with **B** and move the top vertex downwards?
- If you move the top vertex further from the base, as in **C**?
- If you rotate the triangle and deform it as in **D** ?

When lengths of lines and sizes of angles change

- Can all the lengths increase at the same time?
- Can all the angles increase at the same time?
- Can one angle/length change while the other two remain fixed?

If not why not? How could that be discovered?

Explorations of possibilities: blanket, string and toy

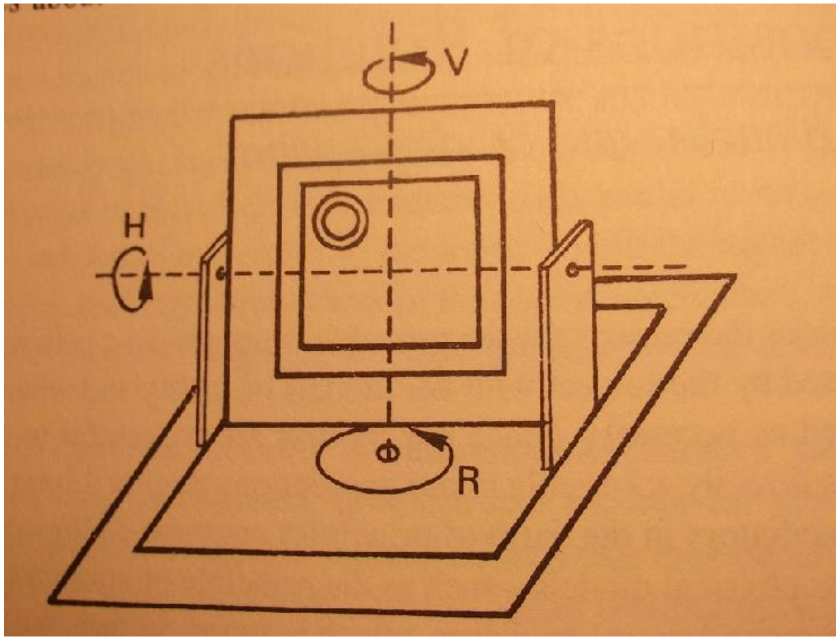


A child, a blanket, a toy
and a string attached to the toy.

This is a domain in which properties of various kinds of matter can make a difference.

- What actions are available to the child?
- What will their consequences be?
- Which ones can bring the toy to the child?
- Which ones can bring the child to the toy?
- What difference does it make if the string is very long?
- What difference does it make if the string is looped round a chair-leg?
- What difference does it make if the string is stretchable?
- What difference does it make if the string is replaced by stiff wire?
- What difference does it make if the toy is on the blanket?
- Or the blanket is replaced by a wooden board with the toy on it?

One of Piaget's examples: Rotating planes



- The central square can rotate about the axis mounted vertically in the outer frame.
- The outer frame can rotate about the axis mounted horizontally across the two pillars.
- The pillars are on a horizontal mount that can rotate about a vertical axis.
- A disc is stuck in the top left quadrant of the central square.

What are the interesting questions that can be asked about

- What processes are possible?
- What states are impossible to achieve from this starting configuration? Why?

Notice that despite the continuous motions this can be treated as a discrete domain: why?

An unexplored domain

I don't know if any child, or teacher has explored the following issues that arise with a set of cubes, or any other collection of items that can be arranged in different configurations on a flat surface.

- Given a collection of blocks, no matter how many there are, they can be arranged to form a straight line – though in some cases walls, furniture or other objects may limit the number that can be included in the line.
- It is also possible to arrange the blocks in several adjacent lines (rows or columns), where adjoining blocks are lined up.

  o o o o o o o o o o

  can be rearranged as

  o o o o o o o
o o o

  or as two lined-up rows:

  o o o o o
o o o o o

  Whereas this cannot be split into two such lined up rows:

  o o o o o o o o o

  But can be split into three lined up rows:

  o o o
o o o
o o o

  A learner who notices this can discover that there are some rows that cannot be split into similar lined up rows, some that can be split into two, some into three, some into four, etc.

  What else could be discovered about regular arrangements of blocks? What difference does it make if the child cannot yet count?

Investigate your own domains

A good way to understand these ideas is giving yourself the experience of explicitly investigating and then trying to systematize a domain, in contrast with doing it implicitly, as in childhood, driven largely by the environment plus genome.

Piaget seems to have noticed that the world is full of “explorable domains”, or “micro-worlds” that one can play with (or play in) then reflect on and build a theory about.

What can you do with one rubber band and a collection of pins and a pin board to hold a stretched band in place. What becomes possible if you add another band, or two, or three?

What can you do with a flat two-sided object on a table top – e.g. a coin or card with differently coloured faces? What else can you do if there are two such objects?
What happens if you add a third, or a fourth ... ?

If you have coins on a chess board what sorts of patterns and pattern-changes can you generate, using different constraints? (e.g. one coin per square, coins only on black squares, only allow vertical and horizontal, or only allow diagonal changes of arrangements.)

What can you do with a long piece of string?

I have been trying to explore a way of thinking about movable (by translating and/or rotation) line-segments in 2-D space to see how much of Euclidean geometry can be reconstructed without making use of the Euclid’s parallel axiom. This is a rather complex exploration domain. See
`http://www.cs.bham.ac.uk/research/projects/cogaff/misc/p-geometry.html`

What can you do with a matchbox, a toothpick and a button?
(I have no idea: I have never tried or thought about it, though as I type this, ideas grow themselves...)

Conjecture: Children (not only humans) spontaneously, or with external encouragement, discover many such exploration domains, and return to some of them repeatedly over weeks, months or years, interleaving and sometimes combining the explorations.

Piaget's experiments on exploration domains

Piaget considered different domains, with different properties, involving

- abstract shapes represented diagrammatically
- physical objects that could be moved (with/without physical constraints)
- liquids and solids
- static structures compared

  Shapes, spatial/topological relationships,
  metrical properties (same or greater length, volume, amount of stuff) etc.
- consequences and limitations of possible changes in a situation
- things changing to achieve a desired result
- questions about what does and does not change (lengths, volumes, amounts of stuff, etc.)
- effects of slope, height and friction on influence of gravity
- Unconstrained consideration of possibilities (what can be done)
- Constrained consideration of possibilities (how to achieve X)
- Constraints from the world: what is impossible
  - Constrained by physics: properties of matter
  - Constrained by geometry: properties of space
  - Constrained by space-time continuity: possible and impossible processes
  - Constrained by topological relations: order, overlap, containment, etc.

I do not know whether Piaget ever described any of his sample domains with mathematical precision, or produced mathematical or computational analyses of their similarities and differences.

A problem with Piaget's experiments

It appears always to be assumed by the experimenters that the children understand the questions they are asked – even if they don't know the answers or don't know how to evaluate and compare answers.

But suppose the process of learning about a new domain is not just
discovering individual facts, and noticing generalisations linking observed facts,

but also
extending the concepts available for formulating questions, factual descriptions, hypotheses, predictions, explanations, desires, intentions, plans, etc., (Sloman, 1978, Ch 2).

Then we need to distinguish (a) experiments probing what the children know from (b) experiments probing what they understand.

However, there are many different ways in which conceptual competences change; and so far we don't have very good theories about that. E.g., there are differences between

- acquiring a new bottom level sub-category in a taxonomy
- adding a new high level branch in a taxonomy
- learning about new relationships (as opposed to categories)
- learning about new kinds of mappings between complex structures (or domains)
- learning new forms of representation supporting concepts and conceptual composition
- learning about new dispositional concepts (fragility, conductance)

  In many cases the new concepts cannot be defined explicitly in terms of the old ones (as in scientific advances). (Sloman, 2007)

Our understanding of mechanisms for supporting such conceptual changes is still very primitive, as shown by the very limited forms of learning in current robots.

Results of the experiments

Unlike much research on children, Piaget's' work does not emphasise statistics of successes and failures, reaction times, times to solve a problem, ages at which various competences are achieved etc.

Statistical presentations (e.g. using averages) can hide important similarities and differences between individual learners, and individual performances of the same learner.

Ignoring those details for the sake of achieving statistical significance is not good science.
(It may sometimes be useful engineering, or medical research.)

In contrast, Piaget and his collaborators probe each child – starting with a standard question or questions and then creatively producing new questions and suggestions on the basis of answers to previous questions.

What their work reveals is that through the whole age range investigated there is wide range of types of

- partial understanding
- misunderstanding
- inability to generalise from specific cases
- over-generalisation from specific cases
- regression from correct to incorrect answers
- use of fallacious reasoning
- construction of bizarre explanations
- inability to combine two or more pieces of information, apparently grasped well
- increases in insight and sophistication over time, though details are different for different children

The results also raise doubts about communication between researchers and children.

Piaget’s conclusions

The books provide many interesting insights into the different sorts of thinking, reasoning, explaining that are required for the various tasks presented, but it is not clear to me that the collection of descriptive and explanatory constructs used by Piaget is adequate, even for his own ultimate explanatory purposes.

Instead he needs a collection of more detailed specifications of changing information-processing architectures and mechanisms, that could be used in a learner’s mind, employing different forms of representation, different ontologies, different algorithms, and different collections of stored information at different times.

The experimental results (and in some cases school examination results), show that the learning is a long, slow, multi-faceted, often idiosyncratic, and usually also erratic process with large variations in results achieved at various ages, or even in a lifetime.

This could lead some to assume that the biological learning mechanisms are deeply flawed and should not be used in machines.

However that ignores the fact that there are there are changes between birth (or conception!) and adult life in humans and many other species that current machine systems do not even begin to match, though in some domains (e.g. some kinds of data-mining) they far outstrip human learners.

Trying to find out what infants can and cannot do at birth (e.g. looking for “core concepts”) is of relatively little interest compared with understanding the processes that produce those various competences (none available at conception!) and continue to develop new ones after birth. (Chappell & Sloman, 2007)

I suspect current theories of learning barely scratch the surface – and fail to specify mechanisms that can account for the changing competences Piaget found, or even the well known patterns of development generally seen in infants and children, and throughout life. So let’s not write-off biology.

Types of situation involving possibility and necessity

We could try to specify in far more detail than ever before (but inspired by Piaget) the many different kinds of competence and knowledge that a typical human acquires, and some of the observed developmental and learning sequences, e.g. learning about:

- Discrete variation (externally, internally imposed boundaries)
- Continuous variation
- Linear structures, 2-D structures, 3-D structures, and spatio-temporal structures
- Properties of materials relevant/irrelevant.
- Only kinematics relevant: or kinematics + dynamics.
- Various kinds of change:
  - translations
  - rotations
  - topological changes
  - shape preserving non-preserving
  - breaking/separating
  - joining/assembling
  - changing information (e.g. visibility, occlusion, rotation)
  - caused by: external processes, viewer motion (translation, rotation)
  - caused by intentional manipulative actions/interactions

    actions by self   actions by others (SHOW ROTATING CUBE)

    effects of inanimate objects

    interactions involving single/multiple contact points

  and many more.

  There are many thousands of things for toddlers to learn.

Questions about Piaget

I do not know the answers to these questions:

Did Piaget ever list systematically, in a generative form, how the various problems he considered within a domain could vary, and why.

(My previous slides merely scratch the surface of such a survey.)

I have the impression he thought about collections of examples and features of the collections, but did not attempt to characterise them as a mathematician (or programmer) might specify the sets in a systematic (generative, axiomatic, programmatic) fashion.

I.e. he did not do in his explicit work, what he must have done as a toddler restructuring vast amounts of piecemeal information into systematically applicable micro-theories – as conjectured below.

Did he ever discuss (explicitly) information processing mechanisms, forms of representation, ontologies, architectures, ... or how these change?

(All of these central to AI/Robotics)

He started moving in the right direction (towards explanatory mechanisms) in considering “conservation” and things that could be expressed using various mathematical ideas (commutativity, associativity, abstraction, invariants).

But did he have a clear vision of where that could lead, e.g. in AI?

Late in life he said he wished he had known about AI as a young researcher
(at a workshop on Genetic Epistemology and AI, in Geneva, circa 1983).

I’ll now move on to a conjecture about how knowledge gained about an exploration domain can change.
(Compare this with what Annette Karmiloff-Smith calls representational re-description?)

Background to a Conjecture

Much knowledge (not all) starts Empirical then becomes proto-mathematical

Language learning: the “U”-shaped curve

- Language starts pattern-based
- Then becomes rule-based
  - When this happens the rules are at first over-generalised causing production errors that were not present previously.
  - This is a temporary phase.
- Then language use becomes rule+pattern based
  - As a result exceptions to the syntactic rules are handled properly.
  - This requires a significant change to the information processing architecture (which would not be easy to program).
- In parallel with that development, mechanisms develop that allow many stored patterns to be retained for rapid comprehension and production (non-trivial extension)
  - This can dominate processing in experts most of the time.
  - (This can also lead language researchers to draw false conclusions about the nature of linguistic competences – e.g. assuming they are inherently statistical.)

The Conjecture (based on work with Jackie Chappell)

The case of language is a later evolutionary development of something much older that has largely gone unnoticed in other animals and pre-verbal children (Sloman, 2008a)

The conjectured genetically determined mechanisms support "layered learning", where what can be learnt in new layers depends on what was previously learnt.

The genetically pre-configured mechanisms evolved in response to very general features of our world, common to explorable domains in many different environments

e.g. different domains including 3-D spatio-temporal structures and processes including topological and geometrical states and processes involving different kinds of stuff (matter) with different (unobservable) capabilities (fragility, elasticity, thermal conductivity, toxicity, etc.)

But what structures the mechanisms actually allow to grow in the individual's information processing architecture can depend on

The morphology and sensorimotor and other mechanisms in the species.

The kind of environment in which exploring and acting occur

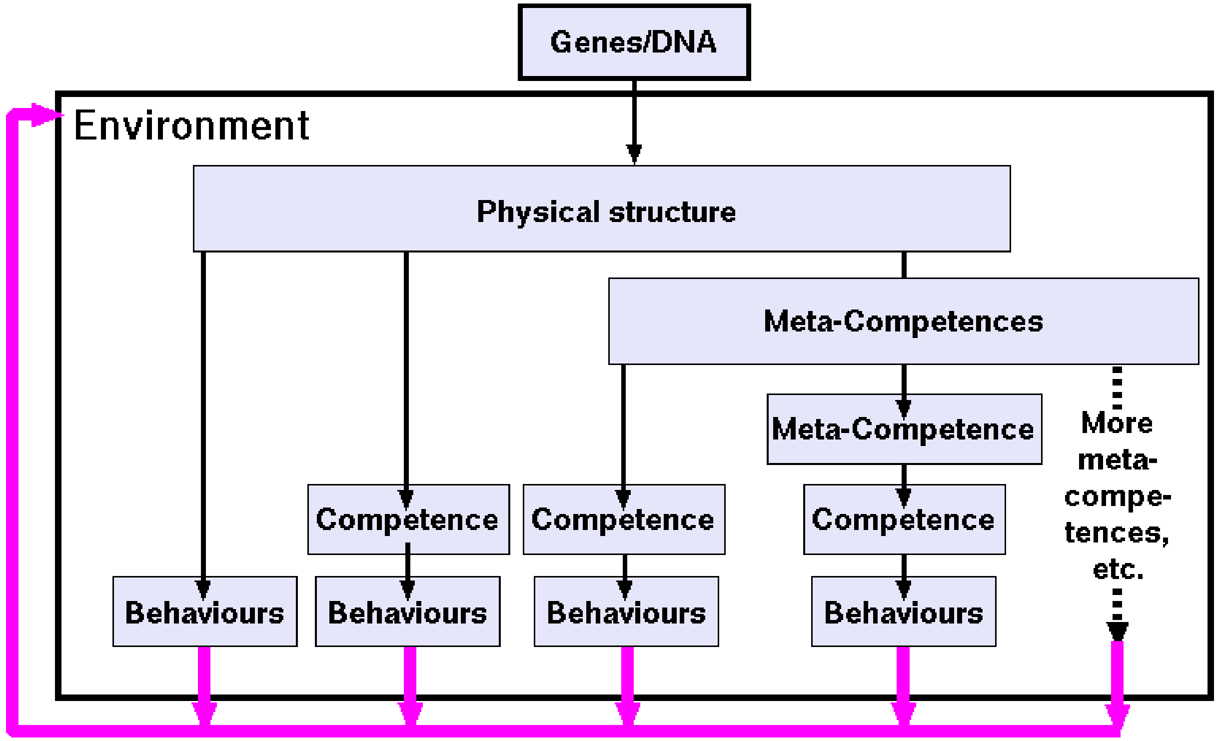
Moreover, I conjecture that, like language learning (which I think evolved later), these more general mechanisms start only pattern-based, then after a while (after enough pattern information has been acquired empirically to be worth compressing) the new meta-cognitive mechanisms attempt to construct a generative/productive theory that can cope with novel situations and generate new patterns.

They don't use a totally general highly optimised set of mechanisms, but satisficing mechanisms (or meta-mechanisms) selected by evolution – to fit a range of space-time environments (McCarthy, 2008).

A diagrammatic summary

Multiple routes from genome to behaviours

(Environment affects all embedded processes)



Work done in collaboration with Jackie Chappell (Chappell & Sloman, 2007)
Chris Miall helped with the diagram.

These ideas are still too vague to be implemented: the point added in this presentation is that many of the competences and metacompetences are concerned with the sorts of exploration domains described above, and ways of (a) combining them into more complex domains (b) finding deeper, hidden, explanatory domains.

The complexity of the developmental processes

Piaget emphasised that there is no fixed linear developmental trajectory

Perhaps because competences in different domains develop in parallel and interact?

Examples of exploration domains that can interact include:

- Learning about aspects of topology and and aspects of geometry can interact: e.g. continuous geometric changes can produce discontinuous topological changes.
- learning about a new type of stuff and how its properties interact with shapes, or other materials
- learning about a new class of actions that can be performed, e.g. sucking, chewing, grasping (with one hand, with two hands, with teeth, with feet), twisting, dropping, throwing, pushing, pulling, ...

  and many more...

Individual developmental trajectories depend on which domains are being explored, what has been learnt in different domains at particular times, which buggy learning has occurred, which domains are combined – all that can vary enormously between learners.

Many partial successes use inadequate solutions – that don't generalise.

Many special cases develop independently before unifying abstractions are discovered.

Applications of what has been learnt in specific situations can generate multiple types of error, often capable of driving new learning (debugging: (Sussman, 1975)).

A very important type of development depends on the fact that many of the domains involve structures and processes in space and time: a fact that allows domains to be combined to generate new, richer, domains – in very many different ways.

(Contrast learning different branches of algebra, or different programming languages.)

Embodiment

In recent years the belief has spread that some older approaches to the study of cognition (e.g. using symbolic forms of representation, such as logic and algorithmic knowledge) are erroneous, because cognition somehow rests on and grows out of features of embodiment including the interactions between an agent's body and the environment

For example, it was found many years ago that giving robots compliant wrists can simplify some manipulation tasks by allowing some of the details of wrist motion to be controlled by interactions between surfaces of objects rather than having to be precisely controlled centrally – e.g. when grasping an object the precise motion of the hand can be controlled directly by the surfaces of the object, rather than by a planning and plan execution sub-system that uses information about the shape of the object.

However, this emphasis on embodiment ignores the fact that there can be structures in the environment that are perceivable and manipulable by animals or robots that have very different morphologies and sensorimotor mechanisms (e.g. birds, primates, elephants).

So biological evolution may well have found a use for methods of learning and solving problems that abstract away from the details of sensorimotor morphology and the detailed dynamics of physical interaction.

This could explain how humans born with various physical deformities, including missing limbs, malfunctioning control systems (palsy), blindness, etc. can develop a rich understanding of the world shared with others.

The strong embodiment thesis also ignores cases of off-line interaction with the environment, in predicting, explaining, planning, describing, narrating, questioning, etc.

I think it is clear that Piaget does not accept the "strong embodiment" thesis.
That's the right response to it! (Sloman, 2009)

The basis of mathematical knowledge

Kenneth Craik suggested that some animals have abilities to “work out” consequences of and constraints on possible actions instead of always having to learn by trial and error (Craik, 1943).

That can be very important when error means death or serious injury – or even just a huge waste of time and energy.

This can be seen as another version of the conjecture that some of the main mechanisms that produce “U-shaped” language learning (starting pattern-based, then develop a more systematic, generative theory usable in novel situations) are involved in forms of learning about domains of exploration, in humans (starting before they learn to talk) and other species.

For more on this see the ideas here about “toddler theorems” and the evolution and development of mathematical competences:

`http://www.cs.bham.ac.uk/research/projects/cogaff/talks/#toddler`

Intelligence, especially mathematical intelligence, largely consists in productive laziness.

Are humans special? How

I suspect that in humans and many other species this form of learning by exploring then reorganising to form a productive system (like going from pattern-based to syntax-based language use) happens in connection with different domains at different times.

However it seems that only humans have the meta-cognitive mechanisms required to explicitly notice, think about, and talk about the kinds of learning they (and the others around them) experience – meta-domains of exploration can also have a developmental trajectory.

A consequence of being such a species is that the use of cultural transmission to accelerate child development can achieve more and more with each successive generation.

But there must be a limit and we may be close to it: the singularity of cognitive catchup, described in
`http://www.cs.bham.ac.uk/research/projects/cogaff/misc/another-singularity.html`

Some questions to be discussed later

1. Can the postulated development from pattern-based to generative information about a domain also be achieved by evolution, or can it happen only during individual development?
2. If it can happen in evolution does that explain some examples of creative problem solving in other animals – using inherited competences?
   (Most animals, e.g. microbes, insects, ... seem to have only inherited competences.)
3. There's lots to be said about how useful, increasingly high level, ontologies can be compiled into the perceptual and action mechanisms of an organism or machine, either by evolution or by individual learning and development.
   (This is a requirement for many innate competences)
   (There are deep implications for the study of vision, for instance.)
4. to be continued ...

To Be Continued

This is a draft set of slides, to be revised and extended as new ideas, or criticisms turn up.

In particular I shall later try to add a section on why it seems likely that young children and many animals do not use a global 3-d cartesian coordinate system to represent the environment and its contents, but instead use a dynamically changing web of **partial** orderings of directions distances, sizes, slopes, curvatures, velocities, accelerations, forces, etc.

Darke (1982) discusses and criticises “The Topological Primacy Thesis”, attributed to Piaget and Inhelder. I don’t think that what I am presenting is the same thesis even though I believe that the sensory and motor resources of biological organisms initially do not have access to a metrically based global coordinate system, and instead have to make do with a complex (and changing) web of partial orderings and topological relationships (including contact, overlap, containment, discontinuity, etc.) The Piagetian experiments described by Darke, are not relevant to testing my hypothesis: it would have to be tested by building and running a working model of the perceptual, learning, reasoning and acting system based on partial orderings, though how to do that is still far from clear.

There’s much more to be said about all this; and much more to be done by roboticists.

I think there are important connections with Karmiloff-Smith’s work on “Relational Redescription”, though I have not yet tried to compare in any detail. Karmiloff-Smith (1992, 1994)

One difference seems to be that K-S specifies that RR involves consciousness, whereas I distinguish the conceptual revision (e.g. from pattern based to syntax-based forms of representation) from the meta-cognitive reflective step of becoming aware of what has been learnt.

I have been concerned with how these processes provide a basis for human mathematics, which links up with Kant’s and Piaget’s ideas about necessity, and with Craik’s theory about use of models.

Luc Beaudoin has pointed me at work by Carl Bereiter, that may be relevant.

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