Meeting of the Aristotelian Society at 21 Bedford Square, London, W.C.1, on Monday, 25th November, 1968, at 7.30 p.m.

IV—EXPLAINING LOGICAL NECESSITY[1]

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Plan: I: Some facts about logical necessity stated. II: Not all necessity is logical. III: The need for an explanation. IV: Formalists attempt unsuccessfully to reduce logic to syntax. V: The no-sense theory of Wittgenstein's *Tractatus* merely reformulates the problem. VI: Crude conventionalism is circular. VII: Extreme conventionalism is more sophisticated. VIII: It yields some important insights. IX: But it ignores the variety of kinds of proof. X: Proofs show *why* things must be so, but different proofs show different things. Hence there can be no *general* explanation of necessity.

I

An adequate theory of meaning and truth must account for the following facts, whose explanation is the topic, though not the aim, of the paper.

(i) Different signs (*e.g.*, in different languages) may express the same proposition.[2]

(ii) The syntactic and semantic rules in virtue of which sentences are able to express contingent propositions also permit the expression of necessary propositions and generate necessary

[1] An early version of this was read to seminars in Hull and Leeds universities in 1963. It was resurrected at short notice and revised, largely under the stimulus of an unpublished paper by Cora Diamond on Wittgenstein's views on necessity.

[2] The best way to identify propositions, for our purposes, is not in terms of truth-conditions but in terms of a *way of identifying* a set of truth-conditions and a set of falsity-conditions. Thus ' My lawn has three straight edges ' and ' My lawn has straight edges meeting in three corners ' both identify the same truth- and falsity- conditions, but in different ways. They express different propositions. Further clarification of this criterion cannot be undertaken here. It is noteworthy that most of the facts about logical necessity discussed herein do not presuppose any asymmetry between truth and falsity. As long as recursive (semantic and syntactic) rules associate with each sentence an empirical procedure for assigning that sentence to one of two categories, all our problems can arise. Compare my " Functions and Rogators " in *Formal Systems and Recursive Functions*, ed. J. N. Crossley and M. A. E. Dummett (North-Holland 1965).

relations between contingent propositions. *E.g.*, although ' It snows in Sydney or it does not snow in Sydney ' can be verified empirically (since showing one disjunct to be true would be an empirical *verification*, just as a proposition of the form ' *p* and not-*p* ' can be *falsified* empirically), nevertheless the empirical enquiry can be short-circuited by showing what the result *must* be.

(iii) At least some such restrictions on truth-values, or *combinations* of truth-values (*e.g.*, when two or more contingent propositions are logically equivalent, or inconsistent, or when one follows from others), result from purely formal, or logical, or topic-neutral features of the construction of the relevant propositions, features which have nothing to do with precisely which concepts occur, or which objects are referred to. Hence we call some propositions logically true, or logically false, and say some inferences are valid in virtue of their logical form, which prevents simultaneous truth of premisses and falsity of conclusion.

(iv) The truth-value-restricting logical forms are systematically inter-related so that the whole infinite class of such forms can be recursively generated from a relatively small subset, as illustrated in axiomatisations of logic.

Subsequent discussion will show these statements to be over-simple. Nevertheless, they will serve to draw attention to the range of facts whose need of explanation is the starting point of this paper. They have deliberately been formulated to allow that there may be cases of non-logical necessity.

II

I think some propositions are necessary though their truth-value is not determined by their logical form alone. For instance, although one can empirically test the proposition that no solid object is completely bounded by three flat sides (*e.g.*, by trying to make one), one can tell in advance what the outcome will be. To prove that no possible state of affairs can make the proposition false, *first* consider the possible ways in which two planes may

be oriented to each other (parallel, or intersecting in a straight line) and *secondly*, for each case consider the possible configurations obtainable by adding a third plane: it is clear that the third cannot close up a space. Such a proof does not make logical deductions from explicit or implicit definitions whose logical structure, but not content, enters into the proof essentially. Rather, it uses essentially the fact that spatial configurations are involved. Admittedly, since we are dealing with empirical concepts allowing borderline cases, we should qualify our theorem thus: There are no *clear* (or *central*) cases of a solid object bounded by exactly three flat sides. (This sort of qualification will be discussed below.) Similarly, there are non-logical proofs of theorems that certain positions cannot be reached in a game of chess played according to the rules, theorems that certain patterns of marks cannot be generated by specified formation rules, and other theorems whose proofs involve not mere logical forms, but also their *contents* in an essential way.

For various reasons (including unwillingness to follow Kant in referring to " intuition ") some philosophers try to reduce all necessity to logical necessity. They would argue that there are relations between the concepts used in our geometrical theorem *without which it is impossible to identify those concepts*, and from which the theorem can be deduced using purely logical steps. Without the italicised condition (or something similar) the claim would be trivial, for whatever theorems we ultimately want to prove could be combined into (perhaps infinitely many) long conjunctions: and from these our theorems could be deduced logically. Modern mathematics is dominated by the abstracting and generalising motive: given a set *S* of theorems on any topic, one can look for a more general theory of which this topic and many others can provide illustrations. This process of abstraction and generalisation can always be continued until a purely logical structure, whose proofs use only logical inference, is reached. As noted already, this is sometimes trivial. However, if the theorems of *S* are not finite in number, and especially if they are not all logically deducible from some finite subset, then the construction of a finitely specifiable axiom system *T* containing analogues of all theorems of *S*, all *logically* deducible within *T*, is not trivial. (In fact, it is *impossible* when *S* contains all theorems of general set theory, or arithmetic.) But even this sort of non-trivial achievement leaves open the question whether there is a way of identifying the concepts of *S* (*e.g.*, ostensively) which neither explicitly nor implicitly uses the axioms of *T* and which nevertheless identifies concepts sufficiently definite to make the theorems of *S* necessarily true. It would then be a non-logical truth that the axioms of *T* define relations which do hold between these concepts or (less plausibly) that the axioms implicitly define concepts necessarily co-extensive with those defined previously. Thus, the empirical concepts used in classifying objects according to shape can be grasped and used without even implicitly using the topic-neutral relations between such concepts which are characterised by the axioms of formal systems of geometry, or at any rate without using enough of them to generate by purely logical inferences such theorems as that no three flat sides can enclose a space.

III

But my main concern is with logical necessity. The question whether all kinds of necessity are logical has been touched on mainly in order to show that it is an open question, and different from the question of the nature of logical necessity, now to be discussed. (Moreover, I think an analysis of non-logical proofs, can shed light on the nature of logical proofs, though there will not be enough space to develop this point here.) The facts about logical necessity referred to in (ii)–(iv) above seem to need some sort of explanation. *How* can its logical form prevent a proposition being falsified by any state of affairs? *How* does the form of an inference prevent the premisses being true and the conclusion false, no matter how things are? It will help to clarify these and similar questions if we start by examining some attempts to give general answers, especially those intended to deflate the questions by showing that there is nothing very remarkable to explain. The motives for such deflation are related to the motives for attempting to reduce all necessity to logic: *e.g.*, abhorrence of special realms of non-contingent fact and peculiar non-empirical means of discovering such facts, and a reluctance to accept anything remotely resembling the thin end of a wedge opening a back door for matephysics. The deflationary strategies we shall examine are formalism, the no-sense theory, and two kinds of conventionalism. My ultimate conclusion is that no general theory explains all kinds of necessity.

IV

Formalism is the attempt to reduce logic to syntax: the structural relations and properties of signs. Its basis is the important fact that a large number of truth- or validity-guaranteeing logical forms can be systematically derived from a subclass (*cf.* (iv), above), and systems of recursively generated symbols can represent these forms. Consequently, by purely symbolic procedures (manipulations of meaningless symbols) we can test for logical truth or validity. The formalist, however, says that we are testing for *nothing but* properties of symbols, and that although we do not explicitly refer to such properties (*e.g.*, derivability in accordance with rules *R* from a set of initial formulae *P*), nevertheless we presuppose the possibility of doing so whenever we infer validly or state something as logically true.[3] It is argued that since logical words " do not refer to anything in the extra-linguistic world ",[4] the rules giving them their meanings must be wholly syntactical, concerned merely with permitted combinations of symbols. The bare bones of the thesis may be obscured by virtuoso constructions of artificial " languages " and the definition of syntactic analogues to such semantic concepts as ' true ', ' denotes ', ' analytic ', *etc.*

The irony is that this reduces logic to geometry. That certain patterns of marks are derivable from certain others by specified manipulations is a geometrical fact whose necessity is at least as much in need of philosophical discussion as logical necessity. There is no possibility of some pattern ceasing to pass the test in a

[3] *E.g.*, see R. Carnap, *Foundations of Logic and Mathematics*, p. 37.

[4] A. M. Quinton, " The *a priori* and the analytic ", *Proc. Arist. Soc.*, Vol. 64 (1963–4), pp. 31–54 (reprinted in *Philosophical Logic*, ed. P. F. Strawson, at p. 123).

strong magnetic field, or on Mars: but why not, and what right have we to be sure? Even if we abstract from notational peculiarities of particular symbolic systems and generalise to the study of structural features common to different systems, we are still in the realm of something more like geometry than logic. For if the symbols are uninterpreted they cannot say anything true or false and our problems about validity and truth do not arise. But if they are interpreted (*e.g.*, correlated by semantic conventions with objects and their properties and relations) then our symbolic tests are tests for something non-syntactical. So if the tests are reliable and never, for instance, select as valid a pattern of inference which is not truth-preserving, then since truth is not a syntactical property of sentences this reliability needs explaining. The explanation will, of course, refer to the connection between formation-rules and truth-condition-rules for the use of topic-neutral signs and constructions (*i.e.*, syntactic and semantic rules). But it will not reduce logic to syntax.

V

The *no-sense* theory (sometimes combined with formalism) attempts to deflate logical necessity by saying that sentences like ' It is now raining outside or it is not now raining outside ' really say nothing, and so cannot be called ' true ' in the usual sense: hence there is no question of *explaining* their necessity. Similarly, in a logically valid inference there is no sense, *i.e.*, nothing said, in the conclusion over and above what is already in the premisses: so the impossibility of true premisses and a false conclusion is simply the trivial impossibility of premisses which are all true yet say something false. There aren't two *distinct* things related so as to prevent one being true and the other false. (Notice that applying this theory to logical relations like contrariety is not quite so straightforward.) A version of this strategy is in Wittgenstein's *Tractatus* (*e.g.*, in 4.46, ff. and 6.1, ff.) along with traces of formalism (*e.g.*, 6.122 and most of 6.126).

In effect, this arbitrarily equates ' being false in no possible states of affairs ' with ' having no sense '. It does not explain why ordinary rules for constructing significant sentences out of meaningful components should sometimes lead to no sense. It rules, apparently without any justification, that our ordinary ways of telling that sentences have different sense and should, for instance, be differently translated into French, simply go wrong when it comes to (*e.g.*) logical truths. However, as noted in (ii), these can sometimes be verified by ordinary empirical procedures. How could one verify something without content? What verifies ' *p* or not-*p* ' empirically need not empirically verify ' *q* or not-*q* ': but how so if they both have the same sense, namely none? Though unnecessary, this is a perfectly ordinary empirical verification (showing one disjunct to be true verifies the whole disjunction). The only difference is that there is a short cut: examination of the logical form shows in advance what the outcome must be (and in some cases it is not quite so obvious). Similarly, we can tell in advance that whatever numbers are substituted for the variables in ' $(a + b)(a - b) + b^2 - a^2$ ', calculation of the result must yield the value 0: nevertheless the laborious calculation is still possible, and not merely in a special sense of ' calculation '. So with verification.

The no-sense theory fails to explain the fact that ordinary empirical procedures of verification sometimes have an outcome which depends on how things are, and can only be determined by going through the procedure, and sometimes do not. It fails to explain how (*e.g.*, with logically related contingent propositions) the outcomes of different verifications cannot vary independently, even though each depends on how things are and may be truth or falsity. It does not explain the difference between cases where logical forms rule out one or other truth-value (or some combinations of truth-values) and cases where something prevents the verification procedure having any outcome at all, as in self-defeating propositions (sometimes called 'category mistakes') like ' Thursdays are three miles long ', or ' The rear axle of France is bald '. Some who quote him with approval were not as aware as Wittgenstein was of all these difficulties, but his only answer seems to have been that some things cannot be asserted, though they show themselves (*e.g.*, *Tractatus* 4.122 and 6.133, f.). We shall return to a similar suggestion later.

VI

The formalist and no-sense theories really do little more than restate some of the facts without offering any explanation. *Conventionalism* attempts to deflate necessity by explaining it as all arising out of arbitrary (not necessarily explicit) decisions to use words in certain ways, or, more vaguely to adopt a certain " logical framework " or " conceptual scheme ". Thus (in *Philosophical Logic*, pp. 11–12) Strawson writes: " Superfacts are seen to be superfluous; and the meanings of sentences expressing necessary propositions are acknowledged . . . to be enough to guarantee their necessity ". Guarantee? How?

Although it may be a matter of convention which rules or interpretations (or logical frameworks, *etc.*) govern the use of our symbols, this does not explain how they can have *consequences*, or *guarantee* anything. Is it a *logical* truth that if these rules or conventions or meanings are associated with symbols then certain combinations of symbols must express true propositions, or valid inferences? Are Strawson's " super-facts " after all lurking in the connexion between rules or conventions and the results they generate? It seems that no complete, non-circular, explanation of logical necessity (*etc.*) can take the form ' All cases of necessity are consequences of so-and-so ' unless ' consequence ' is used in a way which does not imply necessity. *Extreme conventionalism* answers that what the consequences of a convention are is a matter of further convention: this is the most sophisticated deflationary strategy. There are many signs of it in Wittgenstein's later writings[5], and perhaps also in Quine[6], though neither is completely unambiguous.

VII

The main idea behind extreme conventionalism is that there is no way of identifying a rule or convention independently of

[5] *E.g., Philosophical Investigations*, part I, §§ 186, 241–2, 292, 517, and more explicitly in *Remarks on the Foundations of Mathematics, e.g.*, p. 12–13, p. 23, p. 121 ff, though in later passages, *e.g.*, p. 193–6, he retreats to something much more like the *Tractatus* doctrine of " showing ". He seems to have been strongly pulled in several directions.

[6] *E.g.*, " Two dogmas of empiricism " in *Philosophical Review*, 1951, and " Necessary truth " in *The Ways of Paradox*.

explicitly specifying its applications or consequences, yet somehow inexorably determining them *in advance* "like rails invisibly laid to infinity " (*Investigations* I 128). One can formulate an *expression* of the rule in advance of applying it, but what one intends it to mean, how one understands it, is *constituted* by the way one applies it (*e.g.*, in drawing consequences), and therefore cannot explain or justify the applications. If X (partly) constitutes Y, then the existence of Y does not explain X. For instance, someone sitting beside a conveyor belt may have in mind a certain colour or shape and pick up objects coming along the belt if and only if they have the quality he has in mind. It would be natural to say he knows in advance of what actually comes along what procedure or rule he is following, and that his following this rule (having this quality in mind) *explains* his picking up these objects and leaving others. Similarly, we admit that one can grasp a recursive rule for picking out well-formed formulae, or proof-sequences, of a formal system, in advance of actually coming across cases. But our conventionalist (showing behaviouristic and nominalistic tendencies) regards these ways of speaking as misleading insofar as they obscure the fact that the applications one makes are a *criterion* for one's having *this* quality, procedure, or rule in mind rather than some other: they constitute it.

This theory *suggests*, though it cannot consistently *say*, that it is wrong to distinguish sharply between applying a predetermined rule and deciding, or making up one's mind, exactly which rule one is following; or between moves which are right because of a pre-existing decision and moves which we simply decide to call ' right '. It cannot consistently *say* that we are wrong in our use of ' He had no choice but to pick up this one ', ' It follows necessarily that no symbol like this is well-formed ', for this would presuppose an ability to identify what we intend to say independently of what we do say, thus contradicting a basis of the theory.

To get round this difficulty, Wittgenstein tries to show that such concepts as ' necessarily ', ' unavoidably ', ' has to be ', *etc.*, have a rôle in our lives, but not the one philosophers think. For instance, their use can express a certain attitude to some of the decisions we make, such as: the attitude of being unwilling in (almost) any circumstances to revoke the decision, or being resolved to use the decision as some kind of yardstick or standard of comparison for other decisions. Part of what characterises the attitude is the *feeling* that we have no choice, that it is based on some sort of compelling external non-human justification (a " super-fact "?). But our feeling compelled can be explained by such things as: (i) our linguistic training and human nature, which *cause* us to take some decisions rather than others, (ii) our inability to imagine any sort of (*e.g.*, empirical) test showing the decision to be wrong, (iii) the " deep need " that we feel for certain conventions (*Remarks*, p.23) without which we could not have the concepts we do have and communication (with ordinary humans) would break down. Compare someone's *feeling* that his disgust at (*e.g.*) belching after meals is *justified* by the " disgusting nature " of the behaviour, when in fact he has merely been trained to react thus in conformity with a mere convention. His attempts to discuss or argue with someone whose conventions are different may simply break down.

Thus the conventionalist agrees that there are cases of logical necessity, and that rules can *guarantee*, or *determine*, certain consequences, which we can *discover*, for he too feels compelled to take certain decisions and in agreeing that they are not arbitrary he is expressing attitudes we all share. But he stresses that there could be strange creatures who follow our usage so far then suddenly diverge in ways which look wrong to us although they appear to get along smoothly in their own complex social life. We could not understand or explain such behaviour, although we can imagine it happening. We could only draw the conclusion that their concepts (including ' rule ', ' apply ', ' meaning ', ' concept ', ' same ', ' correct ' and ' language ') were different from ours: though to say this is merely to characterise their behaviour not to explain it. Despite our present resolves or attitudes to the contrary, we may ourselves one day come to behave like them, and look back at our previous behaviour with incomprehension. (Compare looking back now at outmoded fashions, or at the code of honour associated with duelling.) Thus there is no absolute non-human justification for the conclusions we actually do draw: that we do is just a fact.

If we object that this conventionalist analysis conflicts with what we mean by ' necessarily ', ' follows logically ', *etc.*, the reply is that what we mean cannot be something over and above the use we actually make of such expressions: and he does not criticise this use, only certain philosophical theories about what lies behind it. This manoeuvre makes it possible for him to agree (in a certain tone of voice) with anything one might say in an attempt to refute him, without allowing that it contradicts anything he is saying. Thus not only does it have the desired effect of deflating necessity, conventionalism also has the apparent advantage of not being directly confrontable with any counter-argument. We shall see, nevertheless, that it is inadequate as an explanatory theory, though it does provide some useful insights.

VIII

If we examine closely some cases of discovering necessary truths which appear to be wholly different from adopting conventions, we shall find hidden complexities. In particular, where we thought we had completely identified some rule independently of its consequences, we might discover a lack of determinateness which could only be removed by taking something like an arbitrary decision to accept or reject an alleged consequence. In this way we find that there is something right about the conventionalist thesis: by getting clear exactly how much is right we are in a better position to point to a residue of error. Our final rejection of conventionalism then is based on its inability to explain adequately the difference between those aspects of (*e.g.*) logical and mathematical discovery which involve deciding to modify concepts and those which do not. A geometrical and a logical example will help to illustrate all this.

Suppose we have proved (cf. section II above) the theorem that no solid is bounded by exactly three flat sides and then come across a large, or small but very thin, steel plate which appears, to the eye, to be bounded by two large triangular surfaces meeting in two sharp edges (like an almost squashed paper cone) and a long thin flat surface joining the remaining two edges. It is tempting to say " The triangular sides cannot be perfectly flat: there must be a gradual curve indiscernible to the eye ": but how can we be sure, without adopting conformity to the theorem as a new criterion for the instantiation of the concepts involved? And how can we be sure that no other unsuspected actual or potential counter-example to the theorem will turn up, unless we choose not to let it?

The history of mathematics shows that however compelling a proof may look it is rash to assume that no counter-example will ever turn up, unless the concepts involved are redefined as suggested above. But it does not follow from this that all a proof does is somehow lead us to such a redefinition: for even if the complex interrelations of our concepts make it difficult for us to survey *all* their possible applications nevertheless the proof may show quite clearly that there is a *range* of cases (*e.g.*, solids bounded by flat sides no two of which are almost superimposed) within which counter-examples are impossible. Although our concepts may, to start with, have unsuspected areas of indeterminateness in which putative counter-examples may turn up, and although within the determinate area there may be configurations not taken into account in the proof, nevertheless in central cases, of a type considered in the proof, the concepts, as identified prior to giving the proof, may be sufficiently determinate to leave no room for further conventions to govern their application. There is much more to be said about this, but first let us look at the logical example, the theorem that all propositions of the form not-(*p* and not-*p*) must be true.

The normal proof starts by assuming that ' not ' and ' and ' are truth-functional connectives defined by the usual truth-tables, and showing how these automatically guarantee that the truth-table for the complex proposition not-(*p* and not-*p*) contains only T's in its final column. A conventionalist might comment that there is nothing which guarantees that every proposition we can express must have a definite truth-value, and a unique truth-value, both of which are assumed in the truth-table proof. Examples using borderline cases of indeterminate concepts, or " category mistakes ", or unsuccessful reference, easily come to mind. Further, cases like ' I am saying something false ', and ' The set of all non-self-containing sets contains itself' might be taken to show that normal procedures for assignment of truth-value sometimes assign both truth and falsity to a proposition. The only thing which can guarantee that these and perhaps other unsuspected types of case will not refute the theorem is a new decision to accept the theorem as giving a criterion for application of such concepts as ' true ', ' false ' and ' proposition ', so that nothing is described as a counter-example. Thus putative counter-examples can be dealt with by not calling them ' propositions ' or by somehow modifying procedures of truth-value assignment. The need for some such decision or convention is especially clear if it is noticed that there are always possibilities of extension of a language, *e.g.*, by introducing new concepts, or new types of use of old concepts and linguistic activities (such as embedding assertions in the context of a new type of ritual); for the only way to be sure that no such extension will generate a new counter-example is to *decide* not to permit any extension unless it preserves the truth of the theorem.

IX

However, all this again ignores the fact that there is a range of central cases where *p* has a determinate truth-value: here there is no possibility of exception, and the truth-table proof shows *why*. Here the central range of cases for which there is no need to adopt a new convention is clearer than in the geometrical example. We thus have two important conventionalist insights: (a) the only way to guarantee that no unsuspected counter-example can turn up is to adopt the truth of the theorem as giving a new criterion for applying the concepts used therein, and (b) there may be great difficulties in specifying precisely the range of central cases for which the theorem and proof hold in their original interpretation, unless it is identified as *the range for which the theorem holds*: yet elaborate proofs hardly seem required for showing that theorems hold where they hold. Despite the importance of these points, however, the conventionalist fails to account for our having a clear view of at least some cases which can be non-circularly characterised and for which the proof demonstrates that they conform to the theorem.

The conventionalist oversimplifies: he asks too sweeping a question (what can guarantee that *nothing* will ever turn up to disprove the theorem?) and gives too sweeping an answer (only a convention to count nothing as a counter-example). But why should there be *infallible* ways of making logical or mathematical discoveries? Why should we always adopt new conventions that what we appear to have proved is to be called ' true ' come what may? (The strains involved in following such a policy through would probably be intolerable.) Why not accept that we can find, through further investigations, that we have made mistakes: that our definitions may lead to borderline cases, or generate inconsistencies; that our proofs concerning their consequences may fail to take account of all cases, or fail to distinguish cases for which different sorts of proof are needed, *etc.*; that precisely what has been proved is not accurately stated in the original theorem; or that we have not provided a non-circular way of identifying the range of cases for which the original theorem is true? But rarely does a proof turn out to have proved *nothing at all*. Moreover, we can give new proofs that our old proofs or theorems were mistaken, and new proofs of the old theorems or new formulations of what the old proofs proved. We can construct new concepts related to the old ones and prove new theorems using them. In all this we can again make mistakes, and discover them in subsequent investigations.[7]

For instance, returning to our logical theorem proved by truth-table, we may examine the cases not covered by the proof and as a result try to give a more general definition of ' proposition ' which assumes only that every proposition identifies a set of truth-conditions and a set of falsity-conditions, allowing that the method of identification may in some cases result in overlapping sets, or non-exhaustive sets (*i.e.*, the proposition lacks a truth-value in some states of affairs). We can then explore different possible ways of redefining the truth-functional connectives and the consequences of the new definitions. Thus ' not ' could be defined as simply interchanging T-conditions and F-conditions, or as producing a new set of T-conditions containing *all* possible states of affairs which are not T-conditions for the original proposition. We could take the F-conditions of ' *p* and *q* ' as the union of the two original sets of F-conditions or as including only conditions where both *p* and *q* have definite truth-values. Alternatively, we can take a narrower definition of ' proposition ', ruling out the cases where T-conditions and F-conditions overlap. General theorems about the consequences of these various moves could then be proved with the aid of modified truth-tables. Or we might prove our old theorem for some restricted class of propositions (*e.g.*, mathematical propositions)—which would involve proving that for *this* class the methods of identifying sets of T-conditions and F-conditions guarantee that each pair of sets is exhaustive and mutually exclusive: for this case the truth-table would merely be the last step in a more complex proof. Thus, what is normally treated as a simple matter in logic books has hidden complexities. (With quantifiers the situation is even more complicated.) Similar explorations, with modified definitions, new sub-ranges, revised formulations of the theorem, *etc.*, are possible with our geometrical example. For instance we could explore the possibility of eliminating the putative counter-example (which must have one long thin side bounded by two edges) by explicitly defining ' flat ' so that two flat surfaces cannot intersect in a curved line and using a theorem that if two lines meet in two points then at least one is curved. Or we may look for a more or less approximate characterisation of the range of cases for which the original proof does work, without requiring any modifications of the concepts involved.

In short, although we *can* adopt new concept-fixing conventions which make all our theorems true by definition (and thus insignificant?), we *need* not: there is a rich realm of possibilities not accounted for by conventionalism, including the possibility of discovering some non-contingent fact about our concepts *as*

[7] I am indebted to I. Lakatos: " Proofs and Refutations " (four parts) in *British Journal for the Philosophy of Science*, 1963.

already identified, such as the fact that some supposed theorem is false. Thus, Formalism, the no-sense theory, and conventionalism each fails to explain or explain away all cases of logical (and non-logical) necessity, though each contains some important insights.

X

Then how are we to explain the facts noted in section I, above? Take as illustration the theorem that if *p* is a proposition with a definite truth-value and ' not ' and ' and ' are defined by the normal truth-tables then the proposition not–(*p* and not-*p*) must be true. (This formulation is immune to the objections raised previously.) If someone asks why, *i.e.*, wants to know the explanation, what can we do but give him a proof? For instance, we show how, on the assumption that *p* has a definite truth-value, T or F, we can use the definitions of ' not ' and ' and ' to construct a truth-table for the complex proposition in which the final column contains nothing but ' T '. Going through the construction shows *why* the rules guarantee this consequence. There is nothing in the construction which allows the time and place or circumstances in which it is carried out to make any difference to the outcome. Hence one can *see* (though this word requires further discussion) that no matter when or where the process is repeated, the result cannot vary in any essential detail. This shows why even if *p* is empirical, and has a truth-value discoverable empirically, the empirical investigation is not necessary in order to show that not-(*p* and not-*p*) is not false. Moreover, by abstracting from the precise features of the notation used, one can see why the result does not depend on which language is used to express the proposition.

Although this example raises more problems than there is time to discuss here, it will suffice to illustrate my thesis that the way to explain a case of necessity is to give a proof of that case. A proof shows *why* a certain logical form guarantees non-falsity, or *why* no three flat sides can enclose a solid (at least in central cases), or *why* a certain position cannot be reached in chess played according to the rules.

The explanation, that is, what the proof shows, is different in different cases. What the proof does, and how, varies enormously. For instance, a proof that a certain kind of thing is possible (*e.g.*, a solid bounded by four flat sides, or a set of ten propositions no two of which can be false simultaneously) works differently from a proof that something is impossible (*e.g.*, a false proposition of the form ' not-(*p* and not-*p*)'). Proving something about a specific type (' No three planes meeting at right angles can enclose a space ') may be different from proving something more general (' No three planes can enclose a space '). Some proofs use " construction lines " (' Suppose we alter the situation thus. . . . ') while others do not. A proof using concepts implicitly or explicitly defined in terms of others works differently from a proof using only ostensively defined concepts. Some proofs use only empirical concepts abstracted from actual instances, while others use *idealised* concepts identified as limiting cases of some sequence. (*E.g.*, the concept of a perfectly straight line, or perfectly flat surface, or the concept of a contingent proposition whose truth-value is definite in all possible circumstances, or the concept of an indefinitely continuing sequence of tosses of a perfectly balanced coin, or the concept of a perfect democracy. A proof using such concepts must show, or assume, that the extrapolation to the limiting case *does* identify a concept. Belief in the absolute correctness of Euclidean geometry, or the applicability of only classical logic in the theory of infinite sets may arise from misplaced confidence in such assumptions.) Some proofs draw attention to something obvious (but perhaps unnoticed), while others reveal hidden connexions.

What is now needed is a detailed and systematic survey of this variety of cases. It would surely undermine such common preconceptions as that every proof proceeds by logical steps, that every proof must be (in principle) checkable by some mechanical procedure, that the purpose of a proof is to provide us with absolute certainty, and that there are some simple, general and basic necessary truths which underlie the rest. Such a survey would put us in a better position to analyse the notion of a proof ' showing *why* such-and-such must be the case '. For the present, we need merely note that since different proofs show different things it is folly to expect one general theory to answer all questions of the form: Why must so-and-so be the case?