Reasoning About Rings and Chains
(Impossible linking and unlinking)

**(CHANGING DRAFT:
Stored copies will become out of date.)**

Aaron Sloman
School of Computer Science
University of Birmingham.

ABSTRACT

This is one of several online discussions of kinds of mathematical competence that seem to have evolved from abilities to perceive and reason about proto-affordances[*] in the environment.

Humans, and possibly other organisms, can use structural relationships to generate a space of possible structures and processes, and then reason about constraints on those possibilities. This seems to require meta-meta-cognitive mathematical reasoning abilities that may not be present in other species, and do not seem to be present in humans at birth. The combined role of the genome and the environment in developing human mathematical competences is discussed in a document on "toddler theorems":
http://www.cs.bham.ac.uk/research/projects/cogaff/misc/toddler-theorems.html
Many of the examples there and in other documents referenced below, are normally thought of as requiring special mathematical training.
However, the ability to understand the impossibility of linking and unlinking two solid rings is sufficiently wide-spread to be the basis of a very common type of conjuring trick used by illusionists: apparently causing two or more rings to become linked then unlinked. The audience does not usually require lessons in topology in order to be amazed at demonstrations like these:
http://www.ellusionist.com/messado-linking-rings-magic.html (Watch the facial expressions)

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[*] *James Gibson's discussions of perception of affordances refer to affordances involving possibilities for and constraints on* ***actions that might be done by the perceiver, and might be relevant to the perceiver's intentions, preferences, or needs****. Perceiving proto-affordances is more basic: it involves seeing possibilities for change in the environment no matter whether the perceiver or any other agent is involved in producing the change, or benefitting or suffering from the change. If Newton really did think about an apple he was thinking about proto-affordances involving the apple and other things including the tree and the ground below. The existence of proto-affordances does not depend on the existence of perceivers.*
For more on vision and language and their evolution see
http://www.cs.bham.ac.uk/research/projects/cogaff/talks/#talk111
What are the functions of vision? How did human language evolve?

This paper
This discussion paper is
http://www.cs.bham.ac.uk/research/projects/cogaff/misc/rings.html

This is closely related to discussions of functions of biological vision in
http://www.cs.bham.ac.uk/research/projects/cogaff/misc/vision
including the role of biological vision in human mathematical discovery,
especially geometry and topology, e.g.
http://www.cs.bham.ac.uk/research/projects/cogaff/misc/triangle-theorem.html

These topics are connected with abilities to perceive and reason about changing affordances:
http://www.cs.bham.ac.uk/research/projects/cogaff/misc/changing-affordances.html

and abilities to perceive some scenes as impossible:
http://www.cs.bham.ac.uk/research/projects/cogaff/misc/impossible.html

A partial index of discussion notes is in
http://www.cs.bham.ac.uk/research/projects/cogaff/misc/AREADME.html
(This is part of the Meta-Morphogenesis project.)

How can you know certain types of deformation of a spatial structure are possible and others impossible?

Impossible 3-D linking/unlinking

Here's an example involving two stone rings forming a chain on an old Indian temple: Vaideshwara temple at Talakad:

FIG 2 Stone rings
From Wikipedia
http://en.wikipedia.org/wiki/Group_of_temples_at_Talakad,_Karnataka
Picture by Hari, Ganesh R, subject to
Creative Commons Attribution-Share Alike 3.0 Unported license.

It appears that each of the rings is made of a single piece of solid stone. Here's a question for the reader
Could the two rings have been cut out of two separate rigid blocks of stone and then assembled?

o How do you know the answer to this?
o Is there any AI/Computer-based reasoning system that could work out the answer?
o Is there any neuroscientific theory that explains how a mathematician's brain makes it possible to work out the answer?

Note (added 14 Aug 2021):
I am grateful to Luc Beaudoin for pointing out that the answer depends on whether blocks of stone can be continuously deformed, e.g. when heated. When I originally posed the question I had implicitly, unwittingly, excluded that possibility, forgetting that the materials of which blocks of stone are made can be deformed if pressure and temperature are sufficiently high, as happens below the surface of this planet! (Though it's unclear that the label "block of stone" would still be applicable in such a context.) Adding "rigid" to the question makes the exclusion explicit. The history of mathematics includes many mistakes arising from such oversights, as illustrated superbly in Lakatos (1976). The possibility of such oversights implies that Kant's claims about the nature of mathematics should not be interpreted as implying that mathematical reasoning is infallible, as pointed out in Sloman (1962).

Making a chain from a set of rings

Creating a chain from a set of disconnected rings is not possible if the rings retain their shape and are made of solid, impermeable material. The start and end states of the impossible transition are depicted crudely below.

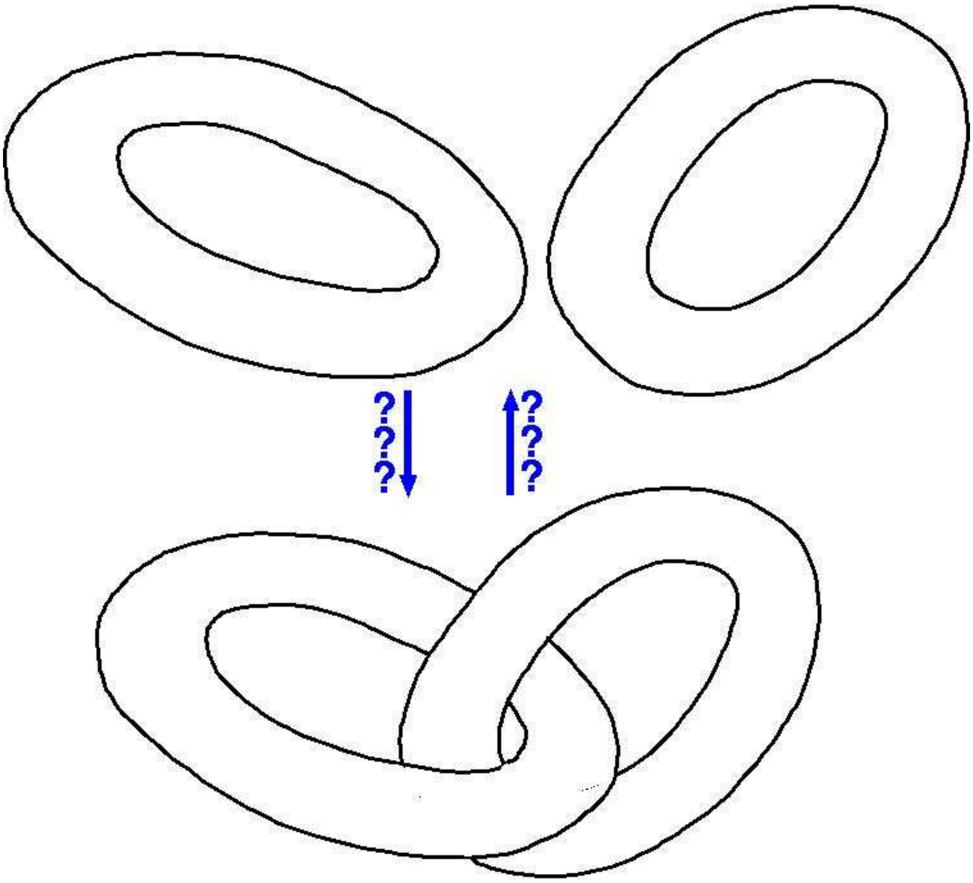


FIG 2 Can two rigid impermeable rings be linked and unlinked?

The answer to the above question seems to require the following reasoning steps.
(a) Making a ring from a block of stone can be done simply by removing parts of the stone without ever reattaching removed parts, and without ever bending or otherwise deforming parts of the stone not yet removed.
(b) If you start with two disconnected blocks of stone and remove material from each you will

always have two disconnected remainders.
(c) If the remainders are rings, and the rings are disconnected they cannot come to be connected (as in a chain) unless either parts are removed and then replaced, or the material of one ring is made to pass through the material of the other ring, which is impossible for two pieces of solid stone.

Added 13 Jun 2017
Is that a different sort of impossibility from the impossibilities concerning linking and unlinking that follow from the impermeability?

Making this argument water-tight is left as an exercise for the reader. As far as I know, no current artificial mathematical reasoner can produce or understand the sort of reasoning used here.

Yet the impossibility of linking solid rings is so obvious to most people that stage conjurers can impress non-mathematical audiences by apparently linking and unlinking rings or closed loops made of impenetrable material, illustrated in these videos:
http://www.ellusionist.com/messado-linking-rings-magic.html (Watch the facial expressions.)

https://www.youtube.com/watch?v=mU3nrRYYIyk
Rings are apparently linked and unlinked (among other things) several times.

https://www.youtube.com/watch?v=-6cxF32ApWM
Separate rope rings become linked: demonstration and tutorial.

Members of the street or auditorium audience do not need to be taught that what they appear to be seeing is impossible. That's not because their mathematics teachers taught them previously. Human mathematical competences are deeper and more widespread than most researchers on human minds realise. (I think Jean Piaget partly understood this, but he did not know how to describe or explain the phenomena. The problems have been ignored by many researchers, including the vast majority of researchers in AI and Robotics.)
NOTE:
Discovering that something is **possible** or **impossible** has nothing to do with probabilities. Possibilities and impossibilities are not points on a scale of probabilities. On the contrary, although it is possible to learn about, think about, and make use of information about what is and is not possible, without making any use of probability, the converse is not true. The idea of a probability value depends on things being possible.

Why are audiences mystified?

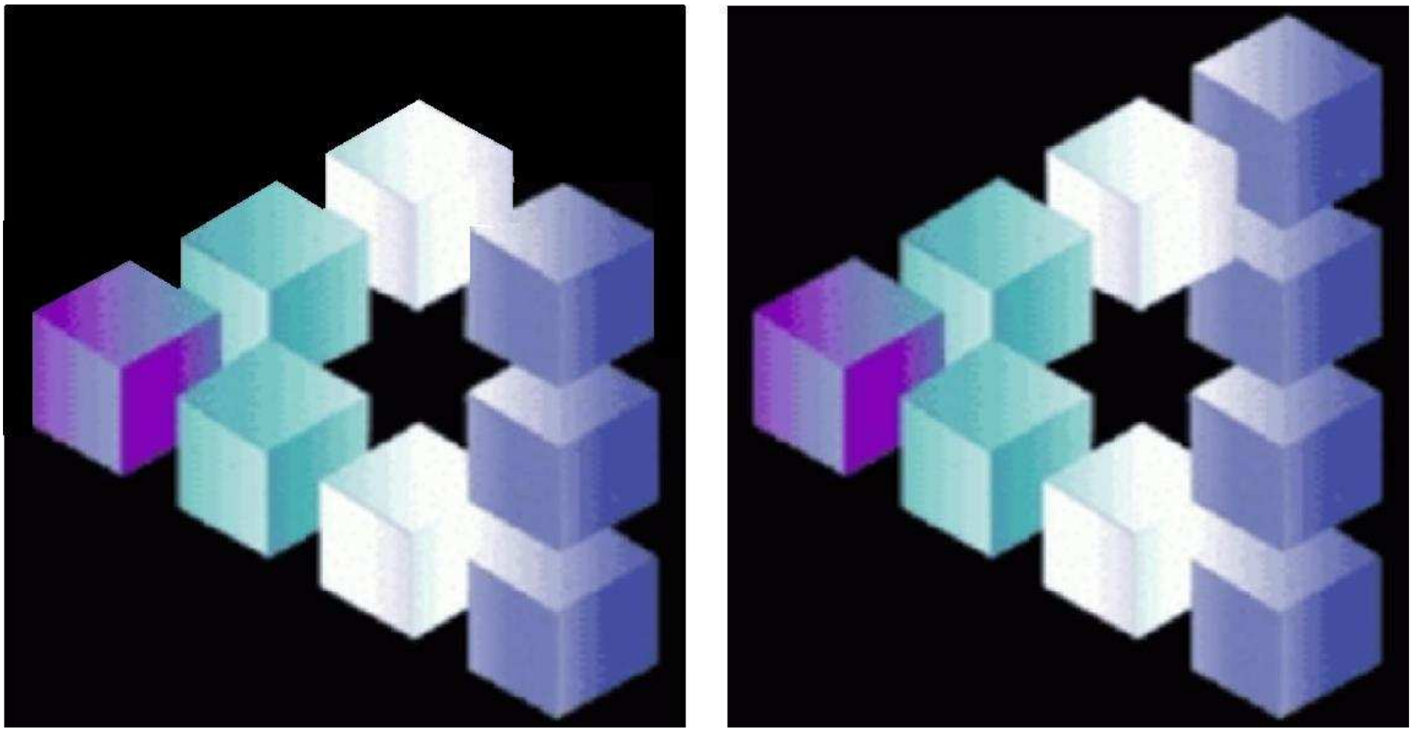
Audiences are familiar with the fact that closed loops or rings can be made from flexible materials, such as ropes or rigid materials such as stone or metal.

They are also aware that two such objects can be moved around relative to each other, but cannot become linked by passing part of the material of one object through the other, if both are made of impenetrable material.

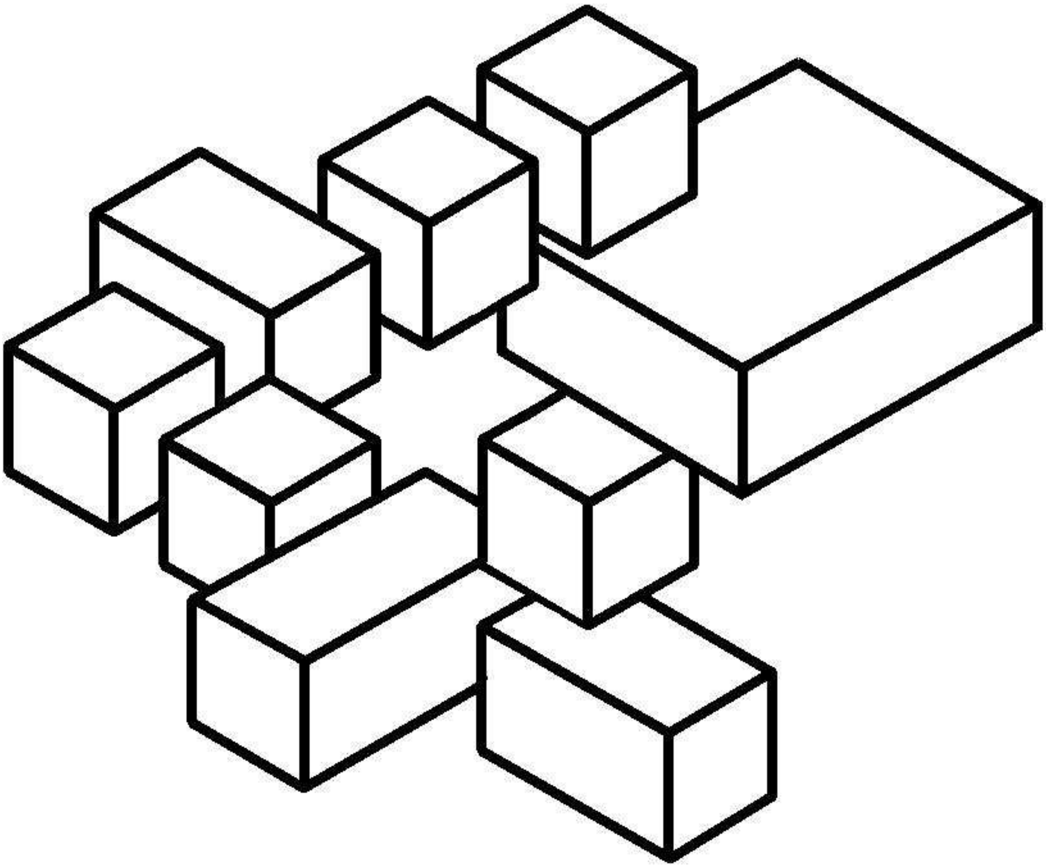
They may be familiar with the fact that knotted ropes form loops that cannot become linked if the knots are not undone. However the conjurer mystifies the audience in this case by leaving a knot partially undone and allowing the configuration of the knot to be transformed in a way that is not obvious to the audience. (Describing the operation in words without the use of a visible demonstration of the process would be a difficult challenge.)

I don't know whether anyone has investigated the range of ages at which children become able to appreciate the trick because they understand the impossibility of what has apparently been demonstrated. I suspect most two-year-olds will be unable to see anything wrong with these linking tricks. The age at which they know something impossible appears to have happened is not interesting. What is interesting is what has to change in their brains or minds (the virtual machinery running on the brains) to enable them to understand the impossibility.

Likewise, very young children may perceive pictures of impossible objects, or impossible configurations of objects, without seeing that what is depicted is impossible.



One of the above configurations is possible, the other impossible.
Based on a picture produced by Oscar Reutersvard in 1934.



It may be a little harder to work out whether this is possible or not.

Understanding impossibilities such as these requires understanding that there are some spatial relationships such as "above" and "further" that are transitive and asymmetric, which many people understand without having learnt the jargon. (Less obviously, there is also a transitive symmetric relationship involved.)

Standard psychological research methodologies make this kind of understanding, or lack of it, hard to investigate, though Jean Piaget devised some interesting experiments, reported in his last two (closely related), posthumously published, books written with collaborators, though I don't think he had good explanatory theories.
Possibility and Necessity, 1983
Vol 1. The role of possibility in cognitive development (1981)
Vol 2. The role of necessity in cognitive development (1983)
University of Minnesota Press, Tr. by Helga Feider from French in 1987

Possibilities and constraints: Curves on a torus

A separate document raises questions of a similar kind, but in relation to continuous deformations of closed curves on the surface of a torus.
http://www.cs.bham.ac.uk/research/projects/cogaff/misc/torus.html

The following document discusses development of the ability to discover "toddler theorems" about what is necessary or impossible in various situations, which I think begins before young children are aware of making those discoveries and in some cases before they can talk about what they understand.

http://www.cs.bham.ac.uk/research/projects/cogaff/misc/toddler-theorems.html

Other examples:
http://www.cs.bham.ac.uk/research/projects/cogaff/misc/shirt.html
http://www.cs.bham.ac.uk/research/projects/cogaff/misc/rubber-bands.html

All these documents are part of the Meta-Morphogenesis project:
http://www.cs.bham.ac.uk/research/projects/cogaff/misc/meta-morphogenesis.html

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Maintained by Aaron Sloman
School of Computer Science
The University of Birmingham